Possibilistic logic representation of preferences: relating prioritized goals and satisfaction levels expressions

Salem Benferhat, Didier Dubois, Souhila Kaci, Henri Prade[1]

Abstract. The preferences of an agent can be expressed in various ways. The agent may indicate goals having different levels of priority for him, or provides classes of choices with their level of satisfaction for him. The first type of specification can be captured in possibilistic logic under the form of constraints on a necessity measure. It is shown in this paper that the second manner for expressing preferences can be encoded as constraints on a so-called "guaranteed possibility" measure (a min-decomposable function with respect to disjunction). The paper shows how each representation is semantically associated with a possibility distribution (which plays the role of a value function), and how necessity-based possibilistic logic representations can be translated directly into a guaranteed possibility-based representations and vice-versa. In logical terms, it corresponds to the transformation of a generalized CNF into a generalized DNF. Reasoning in guaranteed possibility-based logic is also discussed. Moreover, the two types of representations can also be shown to be equivalent to sets of conditional preference statements. Thus, different basic modes of preference expression can be captured in the same framework.

1 Introduction

In decision analysis, preferences are assumed to be represented by a utility or a value function which assesses the degree of satisfaction of each possible choice. However, the end-user of a decision-support system or of a recommender system is not always able to provide such a function directly for expressing preferences. A more implicit specification in terms of constraints may be often more natural for the user. These constraints can have different forms.

The use of possibilistic logic for stating goals with their levels of priority has been advocated by several authors [8, 9]. It has been shown how to recover a qualitative value function from such a specification [6]. However a preference format in terms of prioritized goals is not always the most natural way for expressing what is looked for. Indeed, one may as well indicate that if the choice is taken in some subset, then some level of satisfaction is reached, and this for a collection of subsets. The user may also have comparative statements, specifying for instance that if p is true, having q true is preferred to having q false. This last form of expression has been already related to the possibilistic framework [1]. But, the modelling of constraints, stated in terms of satisfaction levels of subsets of choices, have not been cast in the possibilistic logic setting yet.

The paper shows that this can be done by handling constraints in terms of a so-called guaranteed possibility measure Δ. A constraint of the form $\Delta(\phi) \geq a$ (where ϕ is a logical formula, and a belongs to a linearly ordered scale), means that any solution making ϕ true is at least satisfactory to level a. By contrast, weighted formulas in standard possibilistic logic of the form (ψ, b) are underlied by constraints of the type $N(\psi) \geq b$ where N is a necessity measure. $N(\psi) \geq b$ means that the formula ψ has a priority level at least equal to b, and thus if a choice violates ψ, its satisfaction degree will be upper bounded by $1 - b$ (where $1 - (.)$ is the order-reversing map of the scale). The higher the priority, the smaller the satisfaction degree of any choice violating the constraint. So the greater the number of constraints of the form $N(\psi_i) \geq b_i$, the more restricted is the set of highly satisfactory choices (which may turn to be empty in case of conflict). On the contrary, the greater the number of constraints of the form $\Delta(\phi_j) \geq a_j$, the larger the set of satisfactory choices. The Δ-based information is combined disjunctively, while the N-based information is combined conjunctively. The paper establishes how it is possible to move from a N-based representation to a Δ-based representation and conversely. This greatly facilitates the joint handling of the two types of preference information in the same setting.

The paper is organized as follows. After a short background on standard possibilistic logic, the expression of preferences is discussed in the two basic formats (priority-based vs. satisfaction-based constraints) in Section 3. Section 4 presents the Δ-based logic representation setting and the associated inference machinery, while Section 5 provides the translation of a N-information possibilistic logic base into a Δ-information logic base, and the converse transformation. The concluding remarks point out the interest of the results for representing and fusing information in possibilistic logic, as well as some other uses of the Δ-based logic.

2 Background

We consider a propositional language $\mathcal{L}$ over a finite alphabet $\mathcal{P}$ of atoms. Ω denotes the set of all classical interpretations (called also solutions or choices here). $[\![\psi]\!]$ denotes the set of all models of the proposition ψ.

At the semantic level, the basic notion in possibilistic logic is called a *possibility distribution*, and denoted by π [11]. This is a simple way for encoding a preferential ordering [10]. A possibility distribution π maps each element ω of Ω into the unit interval $[0, 1]$ or more simply in any totally ordered scale (finite or not). Intuitively, a possibility distribution can encode the preferences of an agent among possible choices. $\pi(\omega)$ represents the degree of satisfaction of a choice ω. By convention, $\pi(\omega) = 1$ means that ω is fully satisfactory for the agent, $1 > \pi(\omega) > 0$ means that ω is only somewhat satisfactory, while $\pi(\omega) = 0$ means that ω is not satisfactory at all. When $\pi(\omega) > \pi(\omega')$, ω is preferred to ω'. A possibility distribution π is said to be *normalized* if there exists at least one interpretation ω_0 such that $\pi(\omega_0) = 1$.

A possibility distribution π induces two mappings grading respec-

[1] I.R.I.T. 118 route de Narbonne 31062 Toulouse Cedex 4, France email:{benferhat, dubois, kaci, prade}@irit.fr

tively the consistency and the necessity of a formula:
– The *consistency* or *possibility* degree of ϕ, denoted by $\Pi(\phi)$, expresses to what extent having ϕ true is consistent with the available requirements expressed by the preferences. Formally, $\Pi(\phi)$ is defined by: $\Pi(\phi) = max\{\pi(\omega) : \omega \models \phi\}$.
– The *necessity* (or *priority*) degree of a formula ϕ, denoted by $N(\phi)$, expresses to what extent ϕ is entailed by the prioritized goals. $N(\phi)$ is defined by duality as follows: $N(\phi) = 1 - \Pi(\neg\phi)$.
Namely, $N(\phi) = min\{1 - \pi(\omega) : \omega \not\models \phi\}$.
The duality equation $N(\phi) = 1 - \Pi(\neg\phi)$ extends the existing one in classical logic, where a formula is entailed from a set of propositional formulas if and only if its negation is inconsistent with this set.
A necessity-based possibilistic logic base (a N-information base for short) is composed of a finite set of weighted formulas of the form $\mathbb{G} = \{(\phi_i, a_i) : i \in I\}$, where ϕ_i is a propositional formula and a_i belongs to a priority scale. (ϕ_i, a_i) means that the priority degree of ϕ_i is at least equal to a_i i.e., $N(\phi_i) \geq a_i$. The higher the weight, the more prioritary the goal ϕ_i.
Associated with a N-information base $\mathbb{G}$, is a unique possibility distribution, denoted by $\pi_{\mathbb{G}}$. The interpretations satisfying all the formulas in $\mathbb{G}$ have the highest possibility degree, namely 1, and the other interpretations will be ranked with respect to the highest formula that they falsify, namely we get [6]:

Definition 1 $\forall\omega \in \Omega$,

$$\pi_{\mathbb{G}}(\omega) = \begin{cases} 1 & \textit{if } \forall(\phi_i, a_i) \in \mathbb{G}, \omega \models \phi_i \\ 1 - max\{a_i : (\phi_i, a_i) \in \mathbb{G} \textit{ and } \omega \not\models \phi_i\} & \textit{otherwise.} \end{cases}$$

3 Prioritized goals vs. sets of satisfactory choices

A set of crisp goals with different levels of priority can always be represented as a possibilistic logic base, as illustrated now.

Example 1 *Hierarchical requirements.*
In the database setting [7], requirements of the following form are often considered:
"Property Γ_1 should be satisfied, and among the solutions to Γ_1 (if any) the ones satisfying requirement Γ_2 are preferred, and among satisfying both Γ_1 and Γ_2, those satisfying requirement Γ_3 are preferred and so on".
$\Gamma_1, \Gamma_2, \Gamma_3, \cdots$ *are here supposed to be classical constraints. Thus, one wishes to express that Γ_1 should hold (with importance or priority $a_1 = 1$), and that if Γ_1 holds, Γ_2 should hold with priority a_2, and if Γ_1 and Γ_2 hold, Γ_3 should hold with priority a_3 (with $a_3 < a_2 < a_1$). This can be readily expressed by the possibilistic propositional logic base (where $\Gamma_i = [\![\gamma_i]\!]$)*

$$\mathbb{G} = \{(\gamma_1, 1); (\neg\gamma_1 \vee \gamma_2, a_2); (\neg\gamma_1 \vee \neg\gamma_2 \vee \gamma_3, a_3)\}.$$

A semantically equivalent form for $\mathbb{G}$ [6] can be obtained by applying the possibilistic logic resolution rule, $(\neg\phi \vee \psi, a), (\phi \vee \xi, b) \vdash (\psi \vee \xi, min(a, b))$. Namely $\mathbb{G} = \{(\gamma_1, 1); (\gamma_2, a_2); (\gamma_3, a_3)\}$. It corresponds to the distribution:

$$\pi_{\mathbb{G}}(\omega) = min(\mu_{\Gamma_1}(\omega), max(\mu_{\Gamma_2}(\omega), 1 - a_2)), max(\mu_{\Gamma_3}(\omega), 1 - a_3)) \quad (1)$$

where $\mu_{\Gamma_i}(\omega) = 1$ if $\omega \in \Gamma_i$ and $\mu_{\Gamma_i}(\omega) = 0$ if $\omega \notin \Gamma_i$.
Such an expression, or more generally the expressions obtained with Definition 1, provides conjunctive normal forms (i.e., it is a min of max). They can be turned into disjunctive normal forms (max of min) and then provide a description of the different classes of choices ranked according to their level of satisfaction, as seen in the example below (all the candidates in a class reach the same level of satisfaction).

Example 2 *Let us consider the following two constraint-based evaluation:*
- if ω satisfies A and B, ω is completely satisfactory, and
- if A is not satisfied, solutions should at least satisfy Γ.

Such an evaluation function can be encountered for instance in multiple criteria problems for handling "special" cases (here situations where A is not satisfied) which coexist with normal cases (here situations where both A and B can be satisfied). It can be directly represented at the semantic level by the disjunctive form:

$$\pi_{\mathbb{S}}(\omega) = max(min(\mu_A(\omega), \mu_B(\omega)), min(\mu_\Gamma(\omega), 1 - \mu_A(\omega), 1 - a)) \text{ with } a < 1. \quad (2)$$

The reading of this expression is easy. Either the candidate satisfies both A and B, or if it falsifies A, it satisfies Γ, which is less satisfactory. This expression obtained as the weighted union of the two different classes of more or less acceptable solutions can be transformed into an equivalent conjunctive form like (1); it can be checked that this conjunctive form corresponds to the base $\mathbb{G} = \{(\alpha \vee \gamma, 1), (\neg\alpha \vee \beta, 1), (\alpha, a), (\beta, a)\}$, where A, B, Γ are the sets of models of α, β and γ respectively (Section 5.1 will give a general method for obtaining $\mathbb{G}$ in such a case). This provides a logical, equivalent description of the evaluation process in terms of prioritized requirements to be satisfied by acceptable solutions. Note that in this information base, the formula (β, a) can be removed since it can be recovered from $(\neg\alpha \vee \beta, 1)$ and (α, a) using the possibilistic resolution principle. It is worth noticing that the clausal form corresponding to the possibilistic logic base $\mathbb{G}$ may be sometimes less natural for expressing the goals than the associated normal disjunctive form (2) as shown by Example 2 above. Example 1 illustrates the converse situation where $\mathbb{G}$ provides an easy reaching of the preferences.
The normal disjunctive form provides a logical description of the different subsets of solutions each with their level of acceptability. On the contrary, a possibilistic logic base which can always be put under the form of a conjunction of possibilistic clauses corresponds to a prioritized set of goals.

4 A logical representation of guaranteed possibility measures

4.1 Δ-based information bases

In possibility theory, there is another measure called the *guaranteed possibility* measure [5]:

Definition 2 *The guaranteed possibility measure of ϕ, denoted by $\Delta(\phi)$, is defined by:*

$$\Delta(\phi) = min\{\pi(\omega) : \omega \models \phi\}.$$

Hence $\Delta(\phi \vee \psi) = min(\Delta(\phi), \Delta(\psi))$ and Δ is decreasing w.r.t. logical entailment. $\Delta(\phi)$ estimates the minimal degree of satisfaction of the preferences (encoded by π) when ϕ is known to be true. Clearly, we have $\forall\phi, \Delta(\phi) \leq \Pi(\phi)$; however the guaranteed possibility measure $\Delta(\phi)$ is not related with the necessity measure $N(\phi)$ (although $\Delta(\phi) \leq 1 - N(\neg\phi)$).
This section completes previous works [5] on the logical representation of guaranteed possibility-based logical bases. In particular, we provide the counterparts of subsumption, equivalence between formulas, and inference. More generally, next subsections show that all the notions used in standard possibilistic logic have their "dual counterparts". We start by introducing the notion of Δ-information bases.

A Δ-based information base (Δ-information base for short) is a set of weighted formulas, denoted by $\mathbb{S} = \{[\phi_i, a_i] : i = 1, \cdots, n\}$, where a_i represents the lower bound of the *guaranteed possibility* degree associated with formula ϕ_i i.e. $\Delta(\phi_i) \geq a_i$.

Notation: In the whole paper, formulas in N-information bases are denoted by (ϕ_i, a_i), and those of Δ-information bases are denoted by $[\phi_i, a_i]$. We call the former N-formulas and the latter Δ-formulas.

4.2 From a Δ-information base to a possibility distribution

In Δ-information bases $\mathbb{S} = \{[\phi_i, a_i] : i = 1, \cdots, n\}$, each piece of information $[\phi_i, a_i]$ is viewed as a constraint expressing that any interpretation satisfying ϕ_i is considered as being satisfactory to at least a degree a_i. Therefore, the possibility distribution π associated with $\mathbb{S}$ should satisfy:

$$\forall [\phi_i, a_i] \in \mathbb{S}, \Delta_\pi(\phi_i) \geq a_i,$$

where $\Delta_\pi(\phi_i)$ is the guaranteed possibility degree associated with ϕ_i and computed from π using Definition 2.

Let us first consider a simple case where $\mathbb{S}$ is only composed of one Δ-formula i.e., $\mathbb{S} = \{[\phi_1, a_1]\}$. Figure 1 explicits the set of possibility distributions π satisfying the constraints $\Delta_\pi(\phi_1) \geq a_1$.

Then, it can be easily checked (see Figure 1) that the following possi-

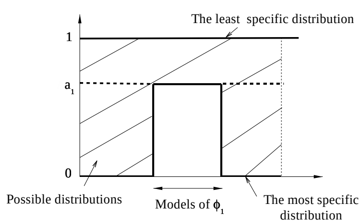

Figure 1. The set of possibility distributions associated with $\mathbb{S} = \{[\phi_1, a_1]\}$.

bility distribution is the smallest, i.e. the most specific one satisfying $\Delta_\pi(\phi_1) \geq a_1$.

$\pi_{\{[\phi_1, a_1]\}}(\omega) = a_1$ if $\omega \models \phi_1$ and $\pi_{\{[\phi_1, a_1]\}}(\omega) = 0$ otherwise.

Namely, the most specific distribution associates the degree a_1 to models of ϕ_1, and the degree 0 to countermodels of ϕ_1.

This remark can be generalized in order to characterize the most specific possibility distribution associated with a Δ-information base. A solution ω is satisfactory to a degree a if the highest degree of the formula satisfied by ω is equal to a, and ω presents no guarantee at all to be satisfactory if it falsifies all formulas of $\mathbb{S}$. More formally,

Definition 3 $\forall \omega \in \Omega$,

$$\pi_{\mathbb{S}}(\omega) = \begin{cases} 0 & \text{if } \forall [\phi_i, a_i] \in \mathbb{S}, \omega \not\models \phi_i \\ max\{a_i : [\phi_i, a_i] \in \mathbb{S} \text{ and } \omega \models \phi_i\} & \text{otherwise.} \end{cases}$$

Note that this definition of a possibility distribution is the dual of the possibility distribution associated with N-information bases. Indeed, when dealing with a N-information base, we are interested in the falsified N-formulas having the highest priority while with a Δ-information base, we are interested in the highest satisfied Δ-formulas. With N-formulas, we apply a minimum specificity principle and we look for the largest possibility distribution agreeing with the constraints. Then we are building the distribution from above by intersecting the elementary distributions $\pi_j(\omega) = 1$ if $\omega \models \psi_j$ and $\pi_j(\omega) = 1 - b_j$ if $\omega \not\models \psi_j$ corresponding to each constraint $N(\psi_j) \geq b_j$. Here with Δ-information, the distribution is built from below by taking the union of the distributions $\pi_{\{[\phi_i, a_i]\}}$.

Example 3 *Let us consider the following Δ-information base. Let $\mathbb{S} = \{[se \wedge su, \frac{1}{4}], [be \wedge se \wedge su, 1]\}$, where the symbols be, se and su stand for "beach", "sea" and "sun" respectively. The first Δ-formula says that the agent is weakly satisfied when there is sea and sun. And the second Δ-formula says that the agent is fully satisfied when moreover there is a beach. The possibility distribution $\pi_{\mathbb{S}}$ associated with $\mathbb{S}$ is the following: $\pi_{\mathbb{S}}(be \wedge se \wedge su) = 1$, $\pi_{\mathbb{S}}(\neg be \wedge se \wedge su) = \frac{1}{4}$ and $\pi_{\mathbb{S}}(\omega) = 0$ for other interpretations.*

The following proposition shows that $\pi_{\mathbb{S}}$ is the most specific possibility distribution:

Proposition 1 *Let $\mathbb{S} = \{[\phi_i, a_i] : i = 1, \cdots, n\}$ be a Δ-information base, and $\pi_{\mathbb{S}}$ be the possibility distribution associated with $\mathbb{S}$ using Definition 3. Then, $\pi_{\mathbb{S}}$ is the unique and most specific possibility distribution satisfying $\Delta(\phi_i) \geq a_i$, for all $[\phi_i, a_i] \in \mathbb{S}$.*

4.3 Semantic equivalence and DNF representation

The semantic equivalence between two Δ-information bases $\mathbb{S}$ and $\mathbb{S}'$ is defined as usual:

Definition 4 *Two Δ-information bases $\mathbb{S}$ and $\mathbb{S}'$ are said to be semantically equivalent iff they generate the same possibility distribution i.e., $\pi_{\mathbb{S}} = \pi_{\mathbb{S}'}$.*

Proposition 2 shows that two Δ-formulas with the same weight in $\mathbb{S}$ can be replaced by their disjunction with also the same weight:

Proposition 2 *Let $\mathbb{S}$ be a Δ-information base and $[\phi, a]$ and $[\psi, a]$ be two Δ-formulas in $\mathbb{S}$. Let $\mathbb{S}' = (\mathbb{S} - \{[\phi, a], [\psi, a]\}) \cup \{[\phi \vee \psi, a]\}$. Then, $\mathbb{S}$ and $\mathbb{S}'$ are semantically equivalent.*

Therefore, each subset of formulas with a given level of priority in $\mathbb{S}$ can be replaced by the disjunction of its formulas. This leads to the Corollary 1 where formulas can be replaced by their DNF form:

Corollary 1 *Let $\mathbb{S}$ be a Δ-information base, and $[\phi, a]$ be a Δ-formula in $\mathbb{S}$. Let $\phi = \{\gamma_1, \cdots, \gamma_n\}$ be the DNF representation of ϕ, namely $\phi \equiv \gamma_1 \vee \cdots \vee \gamma_n$. Let $\mathbb{S}' = (\mathbb{S} - \{[\phi, a]\}) \cup \{[\gamma_1, a], \cdots, [\gamma_n, a]\}$. Then, $\mathbb{S}$ and $\mathbb{S}'$ are semantically equivalent.*

Note that this is the dual of the situation in N-information bases where each N-formula can be replaced by its clausal form (CNF representation). Namely, if $\phi \equiv \delta_1 \wedge \cdots \wedge \delta_m$, then $\mathbb{G}' = \mathbb{G} - \{(\phi, a)\} \cup \{(\delta_1, a), \cdots, (\delta_m, a)\}$ and $\mathbb{G}$ generate the same possibility distribution.

4.4 Subsumption

Within Δ-knowldege bases, subsumed formulas are those that *entail* formulas in the base with a higher satisfaction degree.

Definition 5 *Let $\mathbb{S}$ be a Δ-information base, and $[\phi, a]$ be a Δ-formula in $\mathbb{S}$. Then, $[\phi, a]$ is said to be Δ-subsumed in $\mathbb{S}$ if there exists a formula $[\psi, b]$ in $\mathbb{S}$ such that $b \geq a$ and $\phi \vdash \psi$.*

Indeed, we have the following lemma:

Lemma 1 *Let $\mathbb{S}$ be a Δ-information base, and $[\phi, a]$ be a Δ-subsumed formula in $\mathbb{S}$. Then, $\mathbb{S}$ and $\mathbb{S}' = \mathbb{S} - \{[\phi, a]\}$ are semantically equivalent.*

Example 4 *Let $\{[se \vee \neg be, .9], [se, .5], [be, .4]\}$ be a Δ-information base. It intuitively means that the agent will be satisfied to at least a degree $.4$ if there is a beach, and to at least a degree $.5$ if there is sea, and to at least a degree $.9$ if there is either a sea or no beach (e.g. he is not fond of river beaches!). Clearly, the second formula is subsumed since if there is sea, then the agent will be already satisfied to at least $.9$. The set of all possible interpretations is*
$\Omega = \{\omega_0 : \neg se \wedge \neg be, \omega_1 : \neg se \wedge be, \omega_2 : se \wedge \neg be, \omega_3 : se \wedge be\}$. *Let $\pi_{\mathbb{S}}$ be the possibility distribution associated with $\mathbb{S}$. Then, $\pi_{\mathbb{S}}(\omega_1) = .4$ and $\pi_{\mathbb{S}}(\omega_0) = \pi_{\mathbb{S}}(\omega_2) = \pi_{\mathbb{S}}(\omega_3) = .9$.*
We have $se \vdash se \vee \neg be$ and $\Delta(se) < \Delta(se \vee \neg be)$, then $[se, .5]$ is Δ-subsumed in $\mathbb{S}$. Let $\mathbb{S}' = \mathbb{S} - \{[se, .5]\} = \{[se \vee \neg be, .9], [be, .4]\}$. Then, we can check that $\pi_{\mathbb{S}'}(\omega_0) = \pi_{\mathbb{S}'}(\omega_2) = \pi_{\mathbb{S}'}(\omega_3) = .9$ and $\pi_{\mathbb{S}'}(\omega_1) = .4$. Hence, $\pi_{\mathbb{S}} = \pi_{\mathbb{S}'}$.

The following lemma shows that contradictions are not useful in $\mathbb{S}$ since they do not influence the computation of $\pi_{\mathbb{S}}$, and can be removed without changing $\pi_{\mathbb{S}}$. This is a crucial difference w.r.t. N-based possibilistic logic.

Lemma 2 *Let $[\bot, a]$ be a contradiction formula in $\mathbb{S}$. Then, $\mathbb{S}$ and $\mathbb{S}' = \mathbb{S} - \{[\bot, a]\}$ are equivalent, namely $\pi_{\mathbb{S}} = \pi_{\mathbb{S}'}$.*

This is the dual of N-information bases where tautologies, which are satisfied by all interpretations, can be removed [4]. Be aware that $[\top, a] \in \mathbb{S}$ should not be removed from $\mathbb{S}$. $[\top, a]$ means that, a priori, all solutions are considered as satisfactory to at least a degree a.

4.5 Inference

Inference from a Δ-information base works in a reverse way. Namely, the following resolution principle holds [5]:

$$[\neg\phi \wedge \psi, a], [\phi \wedge \xi, b] \vdash [\psi \wedge \xi, min(a, b)].$$

This expresses that if making $\neg\phi \wedge \psi$ true is at least satisfactory to level a and making $\phi \wedge \xi$ true is at least satisfactory to level b, then realizing $\psi \wedge \xi$ should be satisfactory at least to level $min(a, b)$. As for the possibilistic logic resolution rule in the case of N-based information, this rule can be useful for deriving equivalent forms of Δ-information bases at the syntactic level. For instance, if the agent is satisfied at level a to be at a sea with no beach, and at level b to be on a beach, it should be satisfied at least at level $min(a, b)$ to be at the sea (with or without beach).

5 Bridging N-information bases and Δ-information bases

Since both N-based information and Δ-based information bases are compact representations of the same distribution, the aim of this section is to show how to transform a Δ-information base to a N-information base and conversely.

5.1 From Δ-information to N-information bases

The aim of this section is, given a Δ-information base $\mathbb{S}$, to construct a N-information base $\mathbb{G}$ such that $\mathbb{S}$ and $\mathbb{G}$ induces the same joint distribution i.e.,

$$\pi_{\mathbb{G}} = \pi_{\mathbb{S}},$$

where $\pi_{\mathbb{G}}$ and $\pi_{\mathbb{S}}$ are the possibility distributions associated with $\mathbb{G}$ and $\mathbb{S}$ applying Definitions 1 and 3 respectively.
Let us first consider a Δ-information base $\mathbb{S}$ only composed of one formula i.e., $\mathbb{S} = \{[\phi, a]\}$. The possibility distribution associated with $\mathbb{S}$ is:

$$\forall\omega, \pi_{\mathbb{S}}(\omega) = a \text{ if } \omega \models \phi \text{ and } \pi_{\mathbb{S}}(\omega) = 0 \text{ otherwise.}$$

Note that $\pi_{\mathbb{S}}$ is subnormalized if $a < 1$. This means that $\mathbb{G}$ should be inconsistent to a degree $(1 - a)$, which means that $\mathbb{G}$ should contain $(\bot, 1 - a)$. Moreover, in order to recover that all countermodels of ϕ are impossible, it is enough to have the formula $(\phi, 1)$ in $\mathbb{G}$. Therefore, we can check that the N-information base associated with $\mathbb{S}$ is

$$\mathbb{G} = \{(\phi, 1), (\bot, 1 - a)\}.$$

Now, let us assume that we have two distinct Δ-formulas $\{[\phi_1, a_1], [\phi_2, a_2]\}$ with $a_1 > a_2$. Then, from Definition 3 we have:

$$\forall\omega, \pi_{\mathbb{S}}(\omega) = \begin{cases} a_1 & \text{if } \omega \models \phi_1 \text{ and } \pi_{\mathbb{S}}(\omega) = a_2 \text{ if } \omega \models \neg\phi_1 \wedge \phi_2 \\ 0 & \text{if } \omega \models \neg\phi_1 \wedge \neg\phi_2. \end{cases}$$

Again, if $a_1 < 1$ then $\pi_{\mathbb{S}}$ is subnormalized. We need then to add $(\bot, 1 - a_1)$. To express that models of $\phi_2 \wedge \neg\phi_1$ are possible to a degree a_2, we need to add:

$$(\neg\phi_2 \vee \phi_1, 1 - a_2),$$

and lastly to express that countermodels of ϕ_1 or of ϕ_2 are impossible we add:

$$(\phi_1 \vee \phi_2, 1).$$

Therefore, the N-information base associated with $\mathbb{S} = \{[\phi_1, a_1], [\phi_2, a_2]\}$ is:

$$\mathbb{G} = \{(\phi_1 \vee \phi_2, 1), (\neg\phi_2 \vee \phi_1, 1 - a_2), (\bot, 1 - a_1)\},$$

which is semantically equivalent to
$\{(\phi_1 \vee \phi_2, 1), (\phi_1, 1 - a_2), (\bot, 1 - a_1)\}$.
The following generalizes the previous result:

Definition 6 *Let $\mathbb{S} = \{[\phi_i, a_i] : i = 1, \cdots, n\}$ be a Δ-information base where each level contains one Δ-formula[2], and such that $a_1 > \cdots > a_n$. We let $a_{n+1} = 0$. We associate with $\mathbb{S}$ the following N-information base:*
$\mathbb{G} = \{(\phi_1 \vee \cdots \vee \phi_i, 1 - a_{i+1}) : i = 1, \cdots, n\} \cup \{(\bot, 1 - a_1)\}$.

Then, we have the following proposition:

Proposition 3 *Let $\mathbb{S}$ be a Δ-information base such that each level contains one formula. Let $\mathbb{G}$ be the N-information base constructed from $\mathbb{S}$ following Definition 6. Then, $\mathbb{S}$ and $\mathbb{G}$ are semantically equivalent i.e., $\pi_{\mathbb{S}} = \pi_{\mathbb{G}}$.*

Example 5 *Let us consider the following Δ-information base $\mathbb{S} = \{[be \wedge se \wedge su, 1], [se \wedge su, \frac{1}{2}], [se, \frac{1}{4}]\}$ which means that the agent is weakly satisfied when there is only a sea, he is more satisfied when moreover there is the sun, and he is fully satisfied when there is moreover a beach. The set of possible interpretations is*
$\Omega = \{\omega_0 : \neg be \wedge \neg se \wedge \neg su, \omega_1 : \neg be \wedge \neg se \wedge su, \omega_2 : \neg be \wedge se \wedge \neg su, \omega_3 : \neg be \wedge se \wedge su, \omega_4 : be \wedge \neg se \wedge \neg su, \omega_5 : be \wedge \neg se \wedge su, \omega_6 : be \wedge se \wedge \neg su, \omega_7 : be \wedge se \wedge su\}$.
We have $\pi_{\mathbb{S}}(\omega_0) = \pi_{\mathbb{S}}(\omega_1) = \pi_{\mathbb{S}}(\omega_4) = \pi_{\mathbb{S}}(\omega_5) = 0$,
$\pi_{\mathbb{S}}(\omega_2) = \pi_{\mathbb{S}}(\omega_6) = \frac{1}{4}$, $\pi_{\mathbb{S}}(\omega_3) = \frac{1}{2}$ and $\pi_{\mathbb{S}}(\omega_7) = 1$.
After applying Definition 6, we get:
$\mathbb{G} = \{((be \wedge se \wedge su) \vee (se \wedge su) \vee se, 1), ((be \wedge se \wedge su) \vee (se \wedge su), \frac{3}{4}), (be \wedge se \wedge su, \frac{1}{2}), (\bot, 0)\}$ *which is equivalent to* $\{(se, 1), (su, \frac{3}{4}), (be, \frac{1}{2})\}$, *where the order of priority between the goals is made clear. Indeed, we can check that using Definition 1*
$\pi_{\mathbb{G}}(\omega_7) = 1$, $\pi_{\mathbb{G}}(\omega_0) = \pi_{\mathbb{G}}(\omega_1) = \pi_{\mathbb{G}}(\omega_4) = \pi_{\mathbb{G}}(\omega_5) = 0$,
$\pi_{\mathbb{G}}(\omega_2) = \pi_{\mathbb{G}}(\omega_6) = \frac{1}{4}$ *and* $\pi_{\mathbb{G}}(\omega_3) = \frac{1}{2}$.

5.2 From N-information to Δ-information bases

We now provide the converse transformation. Namely, given a N-information base $\mathbb{G}$, we construct a Δ-information base $\mathbb{S}$ such that

[2] The fact that we assume that each layer is composed of a unique Δ-formula is not a limitation. Indeed, as it is shown in Proposition 2, a set of Δ-formulas having the same weight can equivalently be replaced by a unique Δ-formula, with the same weight and which is composed of their disjunction.

$\mathbb{S}$ and $\mathbb{G}$ are semantically equivalent i.e., $\pi_{\mathbb{C}} = \pi_{\mathbb{S}}$. Let us first consider $\mathbb{G} = \{(\phi, a)\}$ composed of one formula. We have,

$\forall\omega, \pi_{\mathbb{C}}(\omega) = 1$ if $\omega \models \phi$ and $\omega, \pi_{\mathbb{C}}(\omega) = 1 - a$ otherwise.

Note that all interpretations have a possibility degree at least equal to $1-a$. Then, $\mathbb{S}$ should contain the formula $[\top, 1-a]$. Now, in order to recover that models of ϕ have the highest possibility degree namely 1, we add the formula $[\phi, 1]$.

Then, we can easily check, applying Definition 3, that the Δ-information base associated with $\mathbb{G}$ is:

$$\mathbb{S} = \{[\phi, 1], [\top, 1 - a]\}.$$

Let us now consider the case where $\mathbb{G} = \{(\phi_1, a_1), (\phi_2, a_2)\}$ is composed of two distinct N-formulas with $a_1 > a_2$. Then,$\forall\omega$,

$$\pi_{\mathbb{C}}(\omega) = \begin{cases} 1 & \text{if } \omega \models \phi_1 \wedge \phi_2, \pi_{\mathbb{C}}(\omega) = 1 - a_2 \text{ if } \omega \models \phi_1 \wedge \neg\phi_2 \\ 1 - a_1 & \text{if } \omega \not\models \neg\phi_1. \end{cases}$$

Here, all interpretations have a possibility degree at least equal to $1 - a_1$. Then, $\mathbb{S}$ should contain the formula $[\top, 1 - a_1]$.

Now, to ensure that interpretations satisfying $\phi_1 \wedge \phi_2$ get the possibility degree equal to 1, we add the formula $[\phi_1 \wedge \phi_2, 1]$.

Lastly, to recover the fact that interpretations satisfying $\phi_1 \wedge \neg\phi_2$ are possible to a degree $1 - a_2$ we add the formula $[\phi_1 \wedge \neg\phi_2, 1 - a_2]$.

Therefore, the Δ-information base associated with $\mathbb{G}$ is:

$$\mathbb{S} = \{[\phi_1 \wedge \phi_2, 1], [\phi_1 \wedge \neg\phi_2, 1 - a_2], [\top, 1 - a_1]\},$$

which is equivalent to $\{[\phi_1 \wedge \phi_2, 1], [\phi_1, 1 - a_2], [\top, 1 - a_1]\}$.

The proof that $\pi_{\mathbb{S}} = \pi_{\mathbb{C}}$ can be easily checked by applying Definition 3 and Definition 1 respectively on $\mathbb{S}$ and $\mathbb{G}$. The following definition gives the transformation for a general N-information base $\mathbb{G}$:

Definition 7 *Let $\mathbb{G} = \{(\phi_i, a_i) : i = 1, \cdots, n\}$ be a N-information base where each level contains one formula, and such that $a_1 > \cdots > a_n$ and we let $a_{n+1} = 0$. We define from $\mathbb{G}$ a Δ-information base as follows:*

$$\mathbb{S} = \{[\phi_1 \wedge \cdots \wedge \phi_i, 1 - a_{i+1}] : i = 1, \cdots, n\} \cup \{[\top, 1 - a_1]\}.$$

Then, we have the following proposition:

Proposition 4 *Let $\mathbb{G} = \{(\phi_i, a_i) : i = 1, \cdots, n\}$ be a N-information base where each level contains one N-formula (if two formulas have the same weight, we take their conjunction). Let $\mathbb{S}$ be the Δ-information base constructed from $\mathbb{G}$ applying Definition 7. Then, $\mathbb{G}$ and $\mathbb{S}$ are semantically equivalent i.e., $\pi_{\mathbb{C}} = \pi_{\mathbb{S}}$.*

Example 6 *Consider again the N-information base $\mathbb{G}$ computed in Example 5. After applying Definition 7, we get: $\mathbb{S} = \{[se \wedge su \wedge be, 1], [se \wedge su, \frac{1}{2}], [se, \frac{1}{4}], [\top, 0]\}$ which is equivalent to $\{[be \wedge se \wedge su, 1], [se \wedge su, \frac{1}{2}], [se, \frac{1}{4}]\}$. Indeed, we recover the initial Δ-information base $\mathbb{S}$ given in Example 5.*

6 Concluding discussions

The contribution of this paper is twofold. On the one hand, it provides a new compact logical representation of a possibility distribution encoding preferential or plausibility ordering, which can be computationally useful (since it generalizes DNF). It is also interesting for elicitating information, either of the knowledge type or of the preference type. In particular, we have shown that all basic notions of standard possibilistic logic have their counterparts when dealing with Δ-information bases. So possibilistic information can be handled in terms of N-based logic, of Δ-based logic, of Bayesian-like graphical nets, and in terms of comparative constraints, using recents results [1], with bridges between all these representations.

On the other hand, the paper has particularly advocated the use of Δ-based logic for the representation of preferences, since depending on the situations it may be more natural for the agent to express preferences in terms of prioritized goals or in terms of more or less satisfactory sets of solutions. In general, the agent will use both types of expressions. In such a case thanks to the results of Section 5 it will be possible for instance to turn what is expressed as a N-information base into a Δ-information base and to fuse it disjunctively (in agreement with Definition 3) with the rest of the information which is directly expressed in terms of Δ-constraints. We may as well use the N-based representation as a common framework for fusing (this time conjunctively) preferences if it is more suitable. Besides the bridges [1] with the comparative constraints-based representation of the form $\Pi(\phi_i \wedge \psi_i) > \Pi(\phi_i \wedge \neg\psi_i)$ can be also used for putting all the information in the same format, when a part of the preferences are expressed in that way, which may be also very natural (see [3]).

Lastly, guaranteed possibility measures are also very useful for representing "bipolar" preferences. Indeed, it is often useful to distinguish between positive desires and rejected choices or solutions (because they are more or less inacceptable or impossible for the agent). In [2], it has been proposed to use a Δ-based representation for encoding the positive desires and a N-based representation for expressing what is not impossible for the agent. But in this case, we use a *pair* of possibility distributions (rather than one as in this paper) for encoding the two parts of the information with a consistency condition between them since the positive desires of a rational agent should be included inside what he does not reject. The consistency condition is expressed by the constraints $\pi_{\mathbb{S}} \leq \pi_{\mathbb{C}}$. This condition is different if we have two sets of rejected choices $\mathbb{G}$ and $\mathbb{G}'$, where the consistency is expressed by the requirement that $min(\pi_{\mathbb{C}}, \pi_{\mathbb{C}'})$ is normalized. Hence, one should be cautious in the use of consistency condition. Depending on whether we deal with two sets of rejected choices or with one set of rejected choices and one set of positive desires, the problem is different. In particular, if one starts with a Δ-base $\mathbb{S}$ and a N-base $\mathbb{G}$, then even if we use our transformation of $\mathbb{S}$ to an N-base $\mathbb{G}'$, the consistency condition should be $\pi_{\mathbb{C}'} \leq \pi_{\mathbb{C}}$ and not $min(\pi_{\mathbb{C}'}, \pi_{\mathbb{C}})$ to be normalized. For instance, assume that our language contains four interpretations $\{\omega_1, \omega_2, \omega_3, \omega_4\}$, that the agent expresses a positive desire $\mathbb{S} = \{[\phi, 1]\}$ and a constraint $\mathbb{G} = \{(\psi, \alpha)\}$, where $[\![\phi]\!] = \{\omega_1, \omega_2\}$ and $[\![\psi]\!] = \{\omega_2, \omega_3\}$. It can be checked that $\pi_{\mathbb{S}}(\omega_1) = 1 > \pi_{\mathbb{C}}(\omega_1) = 1 - \alpha$. Note that the N-base associated with $\mathbb{S} = \{[\phi, 1]\}$ is simply $\mathbb{G}' = \{(\phi, 1)\}$. Clearly, $min(\pi_{\mathbb{C}}, \pi_{\mathbb{C}'})$ is normalized, but $\pi_{\mathbb{C}'} \leq \pi_{\mathbb{C}}$ does not hold.

REFERENCES

[1] S. Benferhat, D. Dubois, S. Kaci, and H. Prade, 'Bridging logical, comparative and graphical possibilistic representation frameworks.', in *Proc. of the 6th Eur. Conf. on Symbolic and Quantitative Approaches to Reasoning and Uncertainty (ECSQARU'01), 422-431*, (2001).

[2] S. Benferhat, D. Dubois, S. Kaci, and H. Prade, 'Bipolar representation and fusion of preferences in the possibilistic logic framework', in *Proceedings of the eighth International Confenrence on Principle of Knowledge Representation and Reasoning (KR'02)*, (2002).

[3] S. Benferhat, D. Dubois, and H. Prade, 'Towards a possibilistic logic handling of preferences', *Applied Intelligence*, **14(3)**, 303–317, (2001).

[4] S. Benferhat, D. Dubois, H. Prade, and M. Williams, 'A practical approach to fusing and revising prioritized belief bases.', in *Proc. of EPIA 99, LNAI n^o 1695, Springer Verlag*, pp. 222–236, (1999).

[5] D. Dubois, P. Hajek, and H. Prade, 'Knowledge-driven versus data-driven logics', *Journal of Logic, Language, and Inf.*, **9**, 65–89, (2000).

[6] D. Dubois, J. Lang, and H. Prade, 'Possibilistic logic', *In Handbook of Logic in Artificial Intelligence and Logic Programming*, **(D. Gabbay et al., eds, 3, Oxford University Press**, pages 439–513, (1994).

[7] M. Lacroix and P. Lavency, 'Preferences: Putting more knowledge into queries', in *13th Int. Conf. Very Large Data Bases, 215-225*, (1987).

[8] J. Lang, 'Possibilistic logic as a logical framework for min-max discrete optimisation problems and prioritized constraints', in *Inter. Workshop on Fundamentals of Art. Int. Research*, pp. 113–125, (1991).

[9] T. Schiex, 'Possibilistic contraints satisfaction problems or "How to handle soft constraints ?"', in *Proceedings of 15th Conference on Uncertainty in Artificial Intelligence (UAI'92)*, pp. 268–275, (1992).

[10] Y. Shoham, 'Reasoning About Change', in *MIT Press, MA*, (1988).

[11] L. Zadeh, 'Fuzzy sets as a basis for a theory of possibility', *Fuzzy Sets and Systems*, **1**, 3–28, (1978).